Technical Comments

Errata: "Computer Analysis of Asymmetrical Deformation of Orthotropic Shells of Revolution"

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IN the above article replace 1) $C_1^{( )}$ by $C_1^{(2)}$ in the third equation of Eqs. (1); 2) R by R_2 in the first equation of Eqs. (5); 3) $Z_{1_0}^{-1}$ by $Z_{1_1}^{-1}$ in the first and second equations of Eqs. (17); and 4) Ω_i by $\boldsymbol{\Omega}_i$ in the fourth equation of Eqs. (17).

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Comment on Non-Newtonian Flow

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ABLATION materials of greatest practical interest, organic and inorganic glasses, have a strong tendency to non-Newtonian flow behavior. The occurrence of gas bubbles, caused by boiling, degassing, or decomposition of the substrate promotes also non-Newtonian flow properties. Non-Newtonian behavior is related to flow-induced structural changes within the liquid, caused by the orientation or degree of entanglement of very large molecules. In Ref. 1 the Ostwald-de Waele power-law was chosen:

$$\tau = a(du/dy)^n \tag{1}$$

to describe the non-Newtonian properties of the fluid. This law is valid for isothermal conditions, but it is a fact that the zero shear viscosity of practically all substances with non-Newtonian flow behavior is a strong function of the temperature. In all cases where temperature gradients occur throughout the liquid layer, as in ablation, the Ostwald-de Waele power-law will give wrong results. In order to treat the problem of non-Newtonian, nonisothermal flow of liquids, the laws of the temperature and shear rate dependency of liquids must be combined, and the boundary-layer equations must be integrated with this expression for the viscosity. The generally accepted relationship for the temperature dependency of the visocsity is

$$\mu = \exp(A/T - B) \tag{2}$$

This dependency is also true for non-Newtonian liquids as long as the zero shear viscosity is under consideration. The zero shear viscosity will, in the following, be expressed by μ. Eyring[2] and Bueche[3] developed expressions for the shear dependency of non-Newtonian liquids. For not too large deviations from Newtonian flow, they arrive at the following equation:

$$1/\mu_\tau = (1/\mu)(1 + k\tau^2) \tag{3}$$

where μ_τ is the viscosity at the shear stress τ, and k is a constant, determining the degree of deviation from Newtonian flow properties. If k is negative, the fluid is rheopex, and with a positive k the fluid is thixotropic. By writing Eq. (2) in the form

$$\mu/\mu_i = (T/T_i)^{-n} \tag{4}$$

where μ_i is the zero shear viscosity at interface temperature and the power factor for n is defined by $n = A/T$, the expression for the shear and temperature dependencies of the viscosity can be combined to the following expression, which may be handled easily in the subsequent integration process:

$$\frac{1}{\mu_\tau} = \frac{1}{\mu_i}\left(\frac{T}{T_i}\right)^n (1 + k\tau^2) \tag{5}$$

By neglecting the inertia terms, which is always justified for non-Newtonian liquids, the liquid-layer equations near the stagnation point have the form

$$(\partial/\partial x)(ru) + (\partial/\partial y)(rv) = 0 \tag{6a}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left\{\mu_i\left(\frac{T}{T_i}\right)^{-n} \frac{1}{1 + k\tau^2}\frac{\partial u}{\partial y}\right\} \tag{6b}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{6c}$$

The only deviation from the well familiar liquid-layer equations appears in Eq. (6b) where the viscosity coefficient is replaced by a viscosity function $\mu(\tau, T)$. The integration of Eqs. (6a–6c) for the vicinity of the stagnation point, where not only the shear stress but also the pressure gradient contribute to non-Newtonian behavior, is straightforward and leads to the following result:

$$\mu_i v_w^4(v_w - v_i) = \frac{2a\alpha^2}{n^2}v_w^2 - \frac{4b\alpha^3}{n^3}v_w + x^2k\left\{\frac{4a^3\alpha^2}{n^2}v_w^2 - \frac{24a^2b\alpha^3}{n^3}v_w + \frac{72ab^2\alpha^4}{n^4}\right\} \tag{7}$$

v_w is the total ablation velocity, v_i the ablation velocity by evaporation, $a = \partial\tau/\partial x$, $b = \partial^2 p/\partial x^2$, and α is the thermal diffusivity of liquid. For $k = 0$ the result is equivalent to that obtained by Bethe and Adams.[4] Equation (7) shows that for $x = 0$, that is, at the stagnation point, there is no modification of the flow by non-Newtonian behavior, but deviations increase with x^2 in the vicinity of the stagnation point. The value for k changes over several orders of magnitude for various substances from zero to 10^{-2}. In order to treat the ablation behavior of non-Newtonian liquids, the heat-transfer conditions have to be known in addition to Eq. (7). In case the flow of liquids containing gas bubbles has to be treated, k does not, of course, express structural changes within the liquid, but an experimentally determined proforma value for k may be used.

More comprehensive calculations about the ablation of non-Newtonian liquids are now being carried out and will be reported.

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[1] Wells, C. S., "Unsteady boundary-layer flow of a non-Newtonian fluid on a flat plate," AIAA J. 2, 951–952 (1964).

[2] Eyring, H., "Viscosity, plasticity, and diffusion as examples of absolute reaction rates," J. Chem. Phys. 4, 283 (1936).

[3] Bueche, J., "Influence of rate of shear on the apparent viscosity of A-dilute polymer solutions, and B-bulk polymers," J. Chem. Phys. 22, 1570 (1954).

[4] Bethe, H. A. and Adams, M. C., "A theory for the ablation of glassy materials," Avco Research Rept. 38 (November 1958).

Reply by Author to B. Steverding

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THE author is grateful for the opportunity to exchange views on the flow of non-Newtonian fluids with B. Steverding. The paper[1] to which the preceding comments refer contains the solution to a simple problem of two-dimensional, unsteady, isothermal, purely viscous, non-Newtonian boundary-layer flow. The solution was found through a technique for establishing the conditions for similarity (velocity profiles that transform linearly in y). The situation that Steverding is investigating is very interesting and is basically a different problem since the flow is nonisothermal and steady.

The interesting comparison between the two analyses, as the comment points out, is that of the different expressions used to describe the non-Newtonian effect of shear rate. There is, of course, no need to defend the use of the Ostwald-de Waele (power-law) expression, which is phenomenological in nature, as opposed to the Eyring-Bueche expression, which is based on macromolecular considerations. The lack of a theoretical basis for the power-law is conceded initially, and the justification for its use is only in its form, which allows it to be solved explicitly for shear rate. It would seem that the primary consideration in choosing a formulation of the viscous properties for a boundary-layer problem is the accuracy of the formulation in describing the shear rate dependence over the applicable range of flow conditions. In other words, any formulation that describes the measured fluid properties with a prescribed accuracy (and satisfies the mathematical conditions) is appropriate for use in solving a fluid mechanics problem.

As for the temperature dependence of the viscous properties, it is not obvious to the author that the power-law will give wrong results per se for nonisothermal flow, since the coefficient and exponent could be experimentally determined as a function of temperature. The formulations given by Steverding for the temperature-dependence would be particularly interesting, however, if the constants could be predicted theoretically for non-Newtonian fluids, or if the number of empirically determined constants could be reduced.

One question concerning the comment may be worth mentioning for the sake of non-Newtonian fluid nomenclature. The Eyring-Bueche expression is said to provide for rheopectic or thixotropic behavior of viscosity. Thixotropy (or rheopexy) is generally used to describe time-dependent flow effects.[2, 3] Neither the power-law nor the Eyring-Buethe expressions provide for time-dependence. Rather, it appears that (for $k\tau^2 < 1$) the sign of k determines whether the fluid is shear thinning or shear thickening.

It may be of interest to note that the two-dimensional stagnation boundary layer of a power-law non-Newtonian fluid under conditions of steady isothermal flow (considering the convective terms, but not the inertia terms) does give similar solutions.[4] The boundary-layer thickness δ is given by

$$\delta = \eta_\delta \left[\frac{(n+1)(\bar{x})^{n-1}}{2\, R_{0n}\, \alpha^{2-n}}\right]^{1/(n+1)}$$

where the symbols are those defined in Ref. 1. δ is seen to be directly proportional to $(\bar{x})^{(n-1)/(n+1)}$ and inversely proportional to ${R_{0n}}^{1/(n+1)}$. This is compared with δ for stagnation flow of a Newtonian fluid which is not a function of x and is inversely proportional to ${R_{0n}}^{1/2}$.

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References

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Comment on "Conical Shock-Wave Angle"

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SEVERAL approximation formulas for conical shock-wave angles have recently appeared in this Journal.[1, 2] It is the purpose of this note to point out two other equations, both based on the hypersonic similarity rule, which are useful at high values of the hypersonic similarity parameter $M_1 \sin\theta_v$.

Linnell and Bailey[3] first applied the hypersonic similarity law to the Kopal tables[4] and obtained the following formula for the shock-wave half angle θ_w:

$$M_1 \sin\theta_w = 1 - \cos\theta_v + \{1 + [(\gamma + 1)/2]M_1^2 \sin^2\theta_v\}^{1/2} \tag{1}$$

where M_1 is the freestream Mach number, θ_v the cone semi-vertex angle, and γ the ratio of specific heats. For $M_1 \geq 6$ and $\theta_v < 50°$, Eq. (1) agrees to within 5% with the data of Ref. 4. Notice that, for large values of M_1 and small values of $\cos\theta_v$, Eq. (1) approaches Eq. (2) of Ref. 1. However, under these conditions, say for $M_1 \gtrsim 10$ and $\theta_v \gtrsim 40°$, the air behind the shock-wave will become dissociated and these equations will no longer describe the shock geometry correctly. Instead, a second approximation equation derived from the numerical solutions of Ref. 5 can be used.

When the conical flow solutions of Ref. 5 for dissociated air were obtained, it was pleasantly surprising to find that the results correlated extremely well with the hypersonic similarity parameter $M_1 \sin\theta_v$, even more so than for the case of a perfect gas. As a result, the following estimation formula has been useful in predicting values for the shock-wave angle for cases when the air behind the shock is expected to be in dissociation equilibrium and where tabulated values are not available:

$$M_1 \sin\theta_w = (0.39 - 0.01 \log p_1) + (1.03 + 0.005 \log p_1) M_1 \sin\theta_v \tag{2}$$

where p_1 is the ambient pressure in atmospheres.

Equation (2) agrees with the exact numerical calculations of Ref. 5 to within ±1%, for $M_1 \sin\theta_v \geq 6$.

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